EE 435

Lecture 38

Switched-Capacitor Amplifiers and Switched-Capacitor Filters

Review from Last Lecture

How important is statistical analysis?

Example: What will be the yield if two of the 7-bit FLASH ADCs with yields of 98.8% are combined to obtain an 8-bit ADC?





Y_{ADC}=98.8%

 $Y_{ADC} = 1.52 \bullet 10^{-6}$

- The onset of statistically-induced yield loss can be abrupt
- Intuition is not an acceptable substitute to statistical analysis
- Without statistical analysis/simulation there is a high probability that a data converter will be substantially over designed or under designed and neither is acceptable

Review from Last Lecture String DAC Statistical Performance

standard deviation of INL_k assumes a maximum variance at mid-code



It can be shown that INL_k is Gaussian and

(

$$\sigma_{INLk\max} = \sigma_{\frac{R_R}{R_{NOM}}} \frac{\sqrt{N}}{2}$$

Review from Last Lecture

Example 3: What area is needed for obtaining a 99% yield for an 8-bit string DAC and how does that compare to the area required for a 7-bit DAC with the same yield?

For 99% yield $\sigma_{z} = \sigma_{\frac{R}{R_{N}}} \cdot \frac{\sqrt{N}}{2} = \frac{A_{\rho}}{\sqrt{A}} \cdot \frac{\sqrt{N}}{2} = 0.388$ $\frac{A_{\rho}}{\sqrt{A}} \cdot \frac{\sqrt{N}}{2} = 0.388$ $A_{\rho} = 0.1 \mu m \qquad N = 256$ $A = 4.25 \mu m^{2}$

Area doubled because there are twice as many resistors and each is approximately twice as big so by adding 1-bit of resolution, the area went up by approximately a factor of 4



How important is statistical analysis?

- Statistical analysis of data converters is critical
- Some architectures are more sensitive than others to statistical variations in components
- The onset of yield loss due to statistical limitations is generally quite abrupt and can have disastrous effects if not considered as part of the design process

Recall examples where σ_{VOS} =5mV compared with σ_{VOS} =1mV

 Substantially over-designing to avoid concerns about statistical yield loss is not a practical solution since the area penalty, the speed penalty, and the power penalty are generally quite severe

For the effects of local random variations of a parameter X, generally

$$\sigma_X \propto \frac{A_0}{\sqrt{A_C}}$$

where A_C is the area of the matching critical components and A₀ is a process parameter

Some of the most basic and widely used analog circuits



But ratio accuracy can be very good (0.1% or better with good layout and appropriate area)

How bad is the problem?

PROCESS PARAMETERS	N+ACTV P+ACI	V POLY	PLY2 HR	POLY2	MTL1	MTL2	UNITS
Sheet Resistance	82.7 103.2	2 21.7	984	39.7	0.09	0.09	ohms/sq
Contact Resistance	56.2 118.4	14.6		24.0		0.78	ohms
Gate Oxide Thickness	144					ang	strom
PROCESS PARAMETERS	MTL3	<u>N</u> \PLY	N_WE	LL UN	IITS		
Sheet Resistance	0.05	5 824	815	oh	ms/sq		
Contact Resistance	0.78	3		oh	ms		

COMMENTS: N\POLY is N-well under polysilicon.

CAPACITANCE PARAMETERS	N+ACTV	P+ACTV	POLY	POLY2	M1	M2	М3	N WELL	UNITS
Area (substrate)	429	721	82		32	17	10	40	aF/um^2
Area (<u>N+active</u>)			2401		36	16	12		aF/um^2
Area (P+active)			2308						aF/um^2
Area (poly)				864	61	17	9		aF/um^2
Area (poly2)					53				aF/um^2
Area (metall)						34	13		aF/um^2
Area (metal2)							32		aF/um^2
Fringe (substrate)	311	256			74	58	39		aF/um
Fringe (poly)					53	40	28		aF/um
Fringe (metall)						55	32		aF/um
Fringe (metal2)							48		aF/um
Overlap (<u>N+active</u>)			206						aF/um
Overlap (P+active)			278						aF/um

R = 21.7 Ω / \Box and Cd=0.864fF/ μ ²

Both are orders of magnitude unacceptable !

An amplifier alternative ?

Inverting Amplifier

- Capacitor version is area effective and can have very good accuracy
- The node between C₁ and C₂ is a floating node if the Op Amp has a MOS differential pair at the input
- But if we get any charge on the intermediate node there is no way to get it off

An amplifier alternative ?:

 Φ_1 and Φ_2 are nonoverlapping clocks

During Φ_1

 C_1 is charged to V_{IN} and stores charge $Q_1 {=} C_1 V_{\text{IN}}$

 C_F is discharged and $V_{OUT}=0$

During Φ_2

 $C_{\rm 1}$ is discharged but charge is transferred to $C_{\rm F}$

 Q_2 =- Q_1 and V_{OUT} = Q_2/C_F

Substituting for Q_1 we obtain $V_{OUT} = -\frac{C_1}{C_F}V_{IN}$

Serves as a voltage amplifier during ϕ_2

An amplifier alternative !

 $V_{OUT} = -\frac{C_1}{C_T}V_{IN}$

 Φ_1 and Φ_2 are nonoverlapping clocks

- Many applications only need amplifier output at discrete points in time
- Accuracy can be very good
- Area can be very small

But, what about the switches?

Switches for SC Circuits

- Often a single MOS transistor is adequate (either n-ch or p-ch)
- Sometimes need transmission-gate switch (parallel n-ch and p-ch)
- Switches work very well and can be very small but must manage their R_{ON}

Parasitic Capacitances

3 switches and 2 capacitors

10 parasitic capacitances!

Some of parasitic capacitances may be several percent of the size of C_1 and C_F

Parasitic Capacitances

10 parasitic capacitances!

Stray Insensitive SC Amplifiers

Can show that all diffusion parasitic capacitances do not affect gain

Can show that all 14 diffusion parasitic capacitances do not affect gain

Summing amplifier inputs either inverting or noninverting can be easily obtained

Consider the $B_{\!_{\rm C}}$ asic Integrator

Key performance of integrator (and integrator-based filters) is determined by the integrator time constant I_0

Precision of time constants of a filter invariably determined by precision of I₀

Integrator-Based Filters:

$$\frac{V_{OUT}}{V_{IN}} = T(s) = -\frac{1}{R_0 C_1} \frac{s}{s^2 + s \left(\frac{1}{R_0 C_2}\right) + \frac{1}{R_1 R_2 C_1 C_2}}$$

Second-order Bandpass Filter

Denote as a two-integrator-loop structure

Integrator-Based Filters:

Integrator-Based Filters:

Second-order Lowpass Filter

Denote as a two-integrator-loop structure

- Any filter transfer function can be implemented with integrators and summers
- Some of the best known filter structures are based upon integrators and summers
- Accuracy of RC products is critical in the design of good filters

Consider the Basic Integrator

Accurate control of I₀ is required to build good filters !

- 1. Accuracy of R and C difficult to accurately control particularly in integrated applications (often 2 or 3 orders of magnitude to variable)
- 2. Size of R and C unacceptably large if I_0 is in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

Incredible Challenge to Building Filters on Silicon!

Integrator Design Issues

Consider:

Consider the Switched-Capacitor Circuit

Consider the Switched-Capacitor Circuit

Compare the performance of the following two circuits

Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at arbitrary time t_1

For the RC circuit:

Since V_{in} changes slowly assume input is constant over one clock period

$$Q_{RC} \simeq \int_{t_1}^{t_1 + T_{CLK}} \frac{V_{in}(t_1)}{R} dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{t_1 + T_{CLK}} 1 dt$$
$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right]_{t_1}^{T_1 + T_{CLK}} 1 dt$$

Consider the charge transferred to the feedback capacitor for both circuits in an interval of length T_{CLK} at time t_1

For the RC circuit:

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

Observe that a resistor "transfers" charge proportional to $V_{\mbox{\scriptsize in}}$ in a short interval of $T_{\mbox{\scriptsize CLK}}$

$$\mathcal{Q}_{C1} = \mathcal{Q}_{1} \mathcal{V}_{in} \begin{pmatrix} i_1 \\ 2 \end{pmatrix}$$

Since $V_{in}(t)$ is slowly varying

$$Q_{C1}\simeq C_{1}V_{in}\left(t_{1}\right)$$

But this is the charge that will be transferred to C during phase Φ_2

 $Q_{SC} \simeq C_1 V_{in}(t_1)$

Observe that the SC circuit also transfers charge proportional to V_{in} in short intervals of length T_{CLK}

Comparing the two circuits

$$Q_{RC} \simeq \left[\frac{V_{in}(t_1)}{R}\right] T_{CLK}$$

$$Q_{SC} \simeq C_1 V_{in}(t_1)$$

Equating charges since both proportional to $V_{in}(t_1)$

$$C_{1} \approx \left[\frac{1}{R}\right] T_{CLK}$$
$$R_{EQ} \approx \frac{1}{f_{CLK}C_{1}}$$

Observe that a switched-capacitor behaves as a resistor!

This is an interesting observation that was made by Maxwell over 100 years ago but in and of itself was of almost no consequence

Note that large resistors require small capacitors !

This offers potential for overcoming <u>one</u> of the critical challenges for Implementing integrators on silicon at audio frequencies!

Equivalence Between Rapidly Switched Capacitor and Resistor

$$R_{EQ} \simeq \frac{1}{f_{CLK}C_1}$$

This is a frequency referenced filter!

The expressions $S_{\rm c}^{\rm I_0}$ and $~S_{\rm c_1}^{\rm I_0}~$ have the same magnitude as for the RC integrator

- But the ratio of capacitors can be accurately controlled in IC processes (1% to .01% is achievable with careful layout)
- f_{CLK} can be VERY accurately controlled with a crystal (1 part in 10⁶ or better)
- Variability of I_{0eq} is very small

The SC integrator can dramatically reduce the second main concern for building integrated integrators

Consider again the SC integrator

- чС
- 1. Accuracy of R and C difficult to accurately control (often 2 or 3 orders of magnitude to variable)
- 2. Area of R and C too large in audio frequency range (2 or 3 orders of magnitude too large)
- 3. Amplifier GB limits performance

- 1. Accuracy of cap ratio and f_{CLK} very good
- 2. Area of C1 and C not too large
- 3. Amplifier GB limits performance less

C C Vour

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 V_{IN}

- 1. Accuracy of cap ratio and f_{CLK} very good
- 2. Area of C1 and C not too large
- 3. Amplifier GB limits performance less

Observation of Maxwell (and other "Me Too" up until 1977) on equivalence of resistor and switched capacitor had no impact

Two groups independently observed items 1) and 2) in 1976/1977 timeframe and realized that practical implementations on silicon were possible and that is the genius of the concept

Switched Capacitors and the corresponding charge redistribution circuits now used well beyond the SC filter field

Incredible enthusiasm and effort followed for better part of a decade

sC integrator with summing inputs

sC low-pass filter with summing inputs

Consider again the SC integrator

Observe this circuit has considerable parasitics

 $C_{1EQ} = C_1 + C_{s1} + C_{d2} + C_{T1}$

Parasitic capacitors $C_{s1}+C_{d2}+C_{T1}$ difficult to accurately match

- Parasitic capacitors of THIS SC integrator limit performance
- Other SC integrators (discussed later) ofter same benefits but are not affected by parasitic capacitors

Stray insensitive Inverting and Noninverting SC integrators

Stray Insensitive SC Low-Pass Filter with Inverting and Noninverting Inputs

Arbitrary number of inverting and ioninverting Inputs can be added

What if T_{CLK} is not much-much smaller than $T_{SIG}?$ For $T_{CLK}{<<}T_{SIG}$

What if T_{CLK} is not much-much smaller than T_{SIG} ?

For $T_{CLK} < T_{SIG}$

What if T_{CLK} is not much-much smaller than T_{SIG} ?

For T_{CLK}<<T_{SIG}

What if T_{CLK} is not much-much smaller than T_{SIG} ?

For T_{CLK} < T_{SIG} T_{SIG} φ1 φ₂

Considerable change in V(t) in clock period

What if T_{CLK} is not much-much smaller than T_{SIG} ?

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

for any T_{CLK} , characterized in time domain by difference equation

or in frequency domain characterized by transfer function obtained by taking z-transform of the difference equation

$$H(z) = -\frac{\frac{C_1}{z}}{\frac{z}{z-1}}$$

What is really required for building a filter that has high-performance features?

Frequency domain:

Transfer function

 $T(s) = \frac{1}{RCs}$

Time domain:

Differential Equation

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential equation

What is really required for building a filter that has high-performance features?

Time domain:

Differential Equation

Difference Equation

$$V_{OUT}(t) = V_{OUT}(t_0) + \frac{1}{RC} \int_{t_0}^t V_{IN}(\tau) d\tau$$

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

Accurate control of polynomial coefficients in transfer function or accurate control of coefficients in the differential/difference equation

What if T_{CLK} is not much-much smaller than T_{SIG} ?

 $V_{OUT}(nT+T)=V_{OUT}(nT)-(C_1/C)V_{IN}(nT)$

Switched-capacitor circuits have potential for good accuracy and attractive area irrespective of how T_{CLK} relates to T_{SIG}

But good layout techniques and appropriate area need to be allocated to realize this potential !

Consider the following circuit

Termed a flip-around amplifier

Clock signals are complimentary non-overlapping

The flip-around amplifier $_{_{\Phi_2}}$

$$\mathbf{Q}_{1} = \mathbf{C}_{1} \left(\mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$
$$\mathbf{Q}_{2} = \mathbf{C}_{2} \left(\mathbf{V}_{\mathrm{IN}} - \mathbf{V}^{+} \right)$$

The flip-around amplifier

During Φ_2

The flip-around amplifier

$$Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{IN} - V^+) + C_1 (V_{IN} - V_X) = (C_1 + C_2) V_{IN} - C_2 V^+ - C_1 V_X$$

The flip-around amplifier During Φ_2

 $Q_{2F} = Q_2 + Q_{1T} = C_2 (V_{1N} - V^+) + C_1 (V_{1N} - V_X) = (C_1 + C_2) V_{1N} - C_2 V^+ - C_1 V_X$ $V_{C2F} = \frac{Q_{2F}}{C_2} = \left(1 + \frac{C_1}{C_2}\right) V_{1N} - V^+ - \frac{C_1}{C_2} V_X$ $V_{OUTF} = V_{C2F} + V^+ = \left(1 + \frac{C_1}{C_2}\right) V_{1N} - \frac{C_1}{C_2} V_X$

Comparison of Flip Around Amplifier with previous SC amplifier

If $V_X=0$, both have a positive gain but somewhat more gain for a given capacitor ratio for the flip-around structure

In both cases, gain accuracy dependent upon how closely the capacitor ratios can be controlled

One particularly useful application is where want dc gain equal to 2 (1bit/stage pipeline ADC)

Flip-around requires matching two capacitors, other requires ratio matching of two capacitors

Another Flip Around Amplifier

Clock signals are complimentary non-overlapping

Another Flip Around Amplifier

During phase ϕ_1

Assume C_B discharged at start of phase – must verify later

$$\begin{split} \boldsymbol{Q}_{\text{CA1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \\ \boldsymbol{Q}_{\text{CB1}} &= \boldsymbol{C}_{\text{A}}\boldsymbol{V}_{\text{IN}} \end{split}$$

$$V_{\text{OUT}} = -\frac{Q_{\text{CB1}}}{C_{\text{B}}} = -\frac{C_{\text{A}}}{C_{\text{B}}}V_{\text{IN}}$$

Another Flip Around Amplifier

During phase ϕ_2

From phase ϕ_1 $\label{eq:Q_CA1} \begin{aligned} Q_{CA1} &= C_A V_{IN} \\ Q_{CB1} &= C_A V_{IN} \end{aligned}$

$$Q_{CA2} = Q_{CA1} + Q_{CB1}$$
$$Q_{CB2} = 0$$
$$V_{OUT} = -\frac{Q_{CA2}}{C_A}$$
$$V_{CB} = 0$$

Verified that C_B was discharged at the start of phase ϕ_1

$$V_{OUT} = -\frac{C_A V_{IN} + C_A V_{IN}}{C_A} = -2V_{IN}$$

This structure has a gain of 2 independent of any capacitor matching!

Can modify to get noninverting gain and gains of 3, 4, .., without matching requirements

Non-overlapping Clocks

- Essential that the clocks be non-overlapping
- Simple inverter to derive the complimentary clock will not work
- Must guarantee non-overlap in the presence of PVT variations
- In non-demanding speed applications, ϕ_1 and ϕ_2 will have 25% duty cycles

Stay Safe and Stay Healthy !

End of Lecture 38